Detecting exoplanets by spectroscopy

Example of 51 Pegasus b

Context

An exoplanet is a planet that orbits a star other than our Sun. One of the methods used to detect an exoplanet is to measure the star's radial velocity (i.e. its speed relative to the observer) as a function of time with great precision.

The presence of a companion around the star causes the star to move periodically around the centre of gravity of the star-planet system. This is an indirect method, measuring the effect of the presence of a planet on the star's motion. The radial velocity of the star is calculated from the shift in the position of the stellar spectral lines due to the Doppler-Fizeau effect; this is a spectroscopic method.

The first exoplanet orbiting a star similar to our Sun, 51 Peg b, was discovered in 1995 by a team of Swiss researchers using the radial velocity method, also known as Doppler velocimetry.

The aims of this activity are to understand the method of detecting exoplanets by measuring variations in the radial velocity of the star; learn how to deduce certain properties of the planet from these measurements; understand the limitations of this method.

Objectives

The aim of this activity is to understand the method used to detect exoplanets by measuring variations in the radial velocity of the star. It will also show how certain properties of the planet can be deduced from these measurements. Finally, we will discuss the limitations of this method.

This activity will involve

- analysing the spectroscopic data obtained on the star 51 Peg to deduce the orbital period and the maximum amplitude of the star's velocity in its orbit;
- determining the distance of the planet 51 Peg b from its star and its mass from the previous analysis;
- discussing the limitations of the spectroscopic method for detecting exoplanets.

Concepts covered

- Spectroscopy
- Stellar spectrum
- Wavelengths
- Doppler-Fizeau effect
- Kepler's laws
- Law of gravitation
- Double system
- Uniform circular motion
- Sinusoid

Numerical data

(IS international system: metre, kilogram, second)

Physical quantities

- gravitational constant: G = 6.67x10^{-11} IS
- speed of light: c = 299,792,458 m/s

Astrophysical quantities

- mass of the Sun: $M_{\odot}$ = 2x10^{30} kg
- astronomical unit: 1 a.u. = 1.5x10^{11}m
- mass of Jupiter: M_J = 2x10^{27}kg
- period of revolution of Jupiter around the Sun: 12 years
- mass of the Earth: M_E = 6x10^{24}kg
- Saturn's period of revolution around the Sun: 30 years

Lesson plan

1 The star-planet system

In a double system, formed here by a star and its planet, both the two objects orbit around the system's barycentre or “centre of gravity”.

Even if the barycentre is very close to the centre of the star, the star will still be moving periodically. This is manifested by variations in the radial velocity of the star (i.e. the projection of the object's velocity onto the observer’s line of sight, see Figure 1). We will try to detect these periodic variations in the star's radial velocity through the effect they have on its spectrum. When the star is moving around the system’s centre of gravity, its velocity is periodically directed towards us and then away from us. As the star moves towards us, the light is shifted towards the blue, while as it moves away it is shifted towards the red: this is the Doppler-Fizeau effect. We will therefore measure in the spectrum of the star the blue or red shift, Δλ, of the wavelength of spectral lines corresponding to chemical elements whose wavelength is known from laboratory measurements, λ_{lab}. We can then deduce the radial velocity of the star, *v*, with:

$$\frac{\Delta\lambda}{\lambda} = \frac{\lambda_{obs} - \lambda_{lab}}{\lambda_{lab}} = \frac{v}{c}$$

See Figure 1 for a pictorial explanation of the method.

The velocity of the star in its orbit around the centre of gravity takes the form of a sinusoid and is written:

$$v_{star} = V.\sin\left(2\pi\frac{t}{T} + \varphi\right)$$

… where *V* is the amplitude of radial velocity, *t* is time, *T* is the time period of the sinusoid (i.e. the period of revolution of the planet around its star, which is the same as that of the star around the centre of gravity) and φ is the phase.

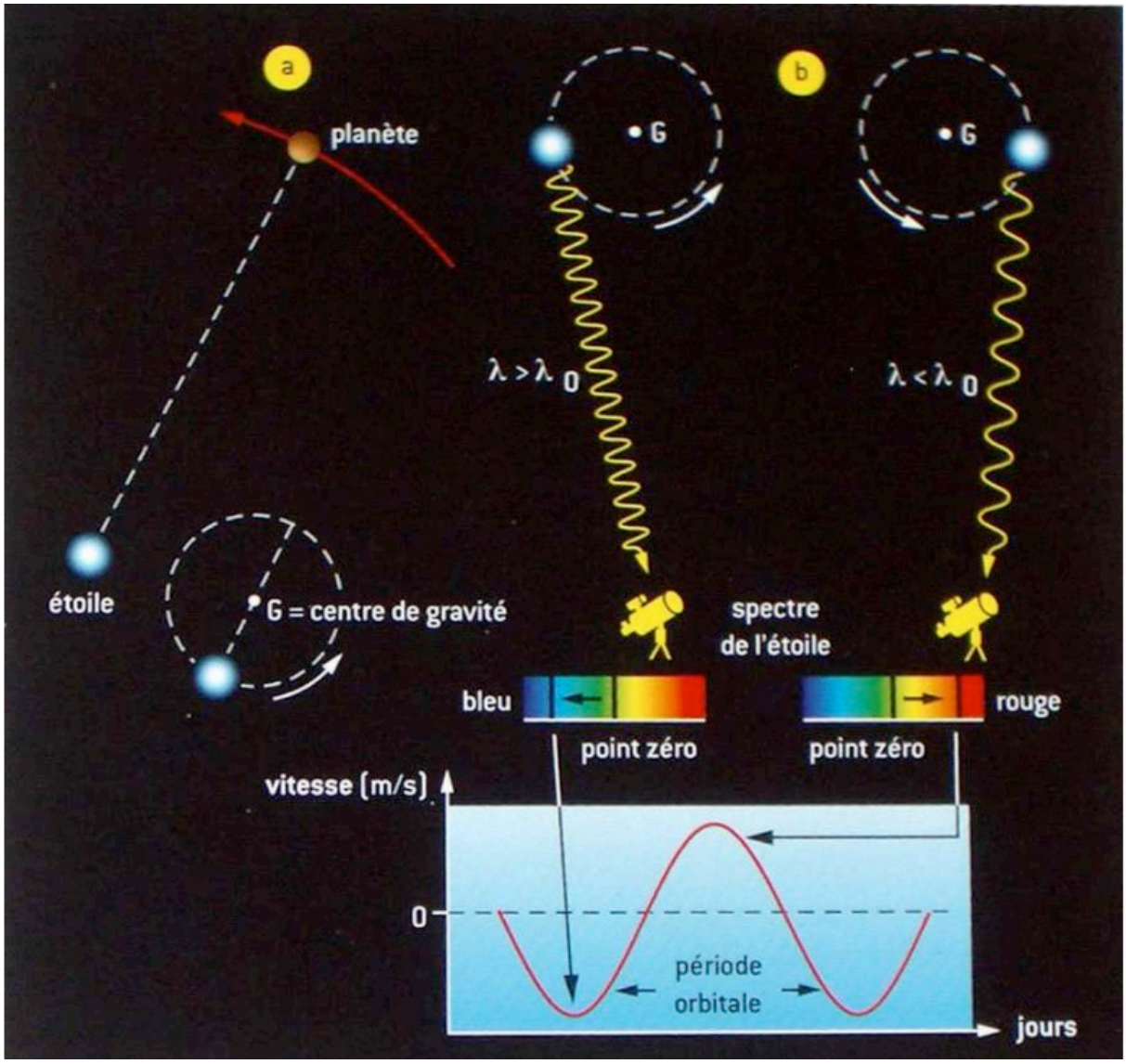

Figure 1: How the spectroscopic method for detecting exoplanets works. (a) The planet revolves around its star; more precisely, the planet and star revolve around G, the centre of gravity of the planet-star system. (b) The Doppler effect enables a terrestrial observer to detect the star's motions. These are generally induced by a sufficiently large planet or group of planets. The succession of red and blue shifts, characteristic of stars influenced by the presence of an orbiting companion, can be plotted as a sinusoidal curve.

Question 1:

On the red curve at the bottom of Figure 1, indicate which parts of this curve correspond to the cases where the star is closest to Earth, where it is furthest away, and the intermediate positions.

2 Analysis of the spectroscopic data that led to the detection of 51 Peg b

51 Peg b is the first exoplanet detected around a solar-type star. It was discovered in 1995 by a team of Swiss researchers, M. Mayor and D. Queloz. They used the ELODIE spectrograph coupled to the 193cm-diameter telescope at the Observatoire de Haute-Provence in France. This enabled them to detect perturbations in the star's trajectory due to the presence of a planet. Figure 2 shows the actual spectrum of the star 51 Peg obtained with the ELODIE spectrograph on 6 November 1995.

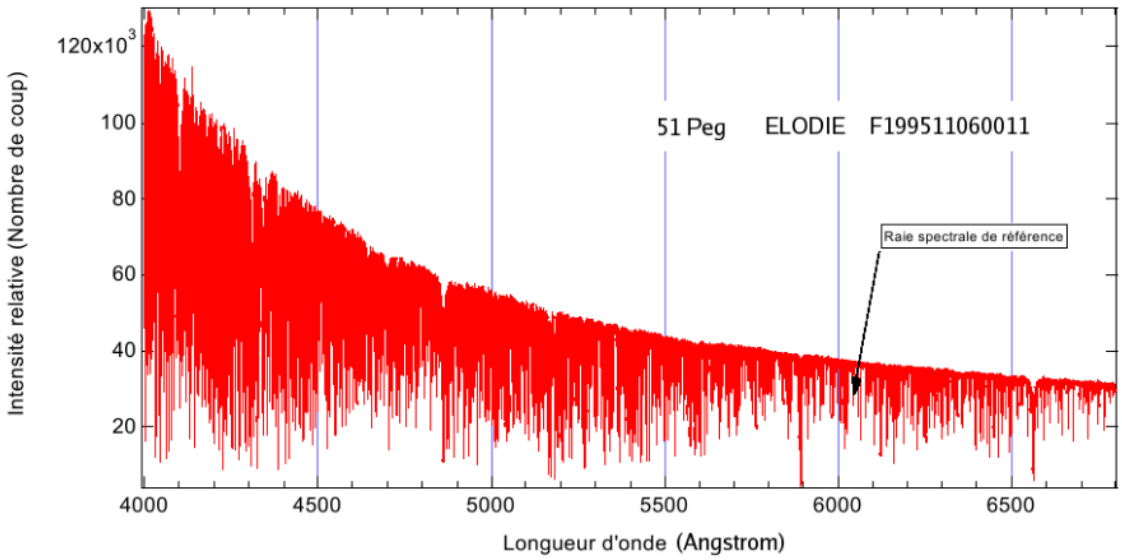

Figure 2: Spectrum of 51 Peg obtained with the ELODIE spectrograph of the 193 cm telescope at the Observatoire de Haute-Provence on 6th November 1995.

Radial velocity variations are calculated using the Doppler shift formula:

$$\frac{\Delta\lambda}{\lambda_{ref}} = \frac{\lambda_{obs} - \lambda_{ref}}{\lambda_{ref}} = \frac{\Delta v_{obs}}{c}$$

λ_{ref} is the wavelength of a line in the spectrum known as the reference line and λ_{obs} is the wavelength of that line in the observed spectrum (λ_{lab} is replaced by λ_{ref} because here we are calculating not the absolute velocity (v) but the velocity variation (Δv)).

To find the variations in the radial velocity of the star due to the presence of a planet, Δv_{obs} is corrected by the radial velocity due to the rotation of the Earth around its axis and around the Sun projected in the direction of the star, noted v_{Earth}:

$\Delta v = \Delta v_{obs} - v_{Earth}$

v_{Earth} is of the order of 20 km/s.

Δv is calculated for each line in the spectrum. The radial velocity variations of all the lines are then averaged. The radial velocity variations thus obtained ($< \Delta v >$) on the star 51 Peg by two American astronomers, Butler & Marcy (results similar to those of Mayor & Queloz) are shown in Table 1.

Questions:

1. Draw the curve of radial velocity variations, $< \Delta v >$, as a function of time in days in Figure 3 from the data in Table 1.
2. Plot the points of this curve on tracing paper.
3. Compare the curve on the tracing paper to the two curves representing two different sinusoids (Figure 4 and Figure 5). Which one best represents the data?
4. Measure the period, T (in days), and the amplitude, K (in m/s), of the sinusoid that best represents the data.

Time *t* (days)	Velocity < Δ*v* > (m/s)		Time *t* (days)	Velocity < Δ*v* > (m/s)
2.67365	-40.87		23.6448	-37.30
2.80873	-46.96		23.6904	-31.35
2.95979	-47.39		23.7258	-42.57
3.62695	-19.85		24.6391	-33.47
3.73265	-13.47		24.7261	-27.46
3.90077	-5.782		24.8174	-22.70
4.60753	49.41		25.6226	45.34
4.78429	58.66		25.7145	47.61
4.90699	63.20		25.7615	56.15
5.60503	45.60		26.6176	65.32
5.92544	17.88		26.6508	69.63
11.6443	-45.50		26.7020	62.54
11.8378	-38.98		26.7366	50.86
12.6355	27.27		27.6474	-22.66
12.8664	32.45		27.6893	-22.55
13.6242	63.38		27.7399	-31.82
13.8295	54.78		27.7721	-31.72
14.6428	-1.340		28.6061	-44.12
14.7229	-5.477		28.6556	-33.59
14.8161	-10.73		28.7019	-37.10
14.9043	-26.29		28.7450	-39.13
15.6261	-50.68		28.7799	-35.32
15.7467	-45.78		29.6093	25.09
15.8654	-57.54		29.7001	35.69
16.6149	10.56		29.7468	41.17
16.7592	12.91		30.6004	61.33
16.8474	16.40		30.6454	58.98
17.7341	63.64		30.6930	63.07
17.8442	60.48		30.7402	56.86
18.6161	26.29		30.8334	50.98
18.7648	5.988		31.6619	-2.545
18.8540	-7.798		31.7052	0.7633
19.6218	-43.62		31.7492	-4.599
19.7325	-59.45		32.6045	-38.53
19.8473	-58.60		32.6479	-48.68
20.6137	-20.22		32.6943	-38.84
20.7401	-8.064		33.6073	2.696
20.8543	5.596		33.6527	11.46
21.6151	56.41		33.6982	14.02
21.6620	59.26		33.7331	17.55
21.7063	66.78		35.5970	19.74
23.5986	-35.13			

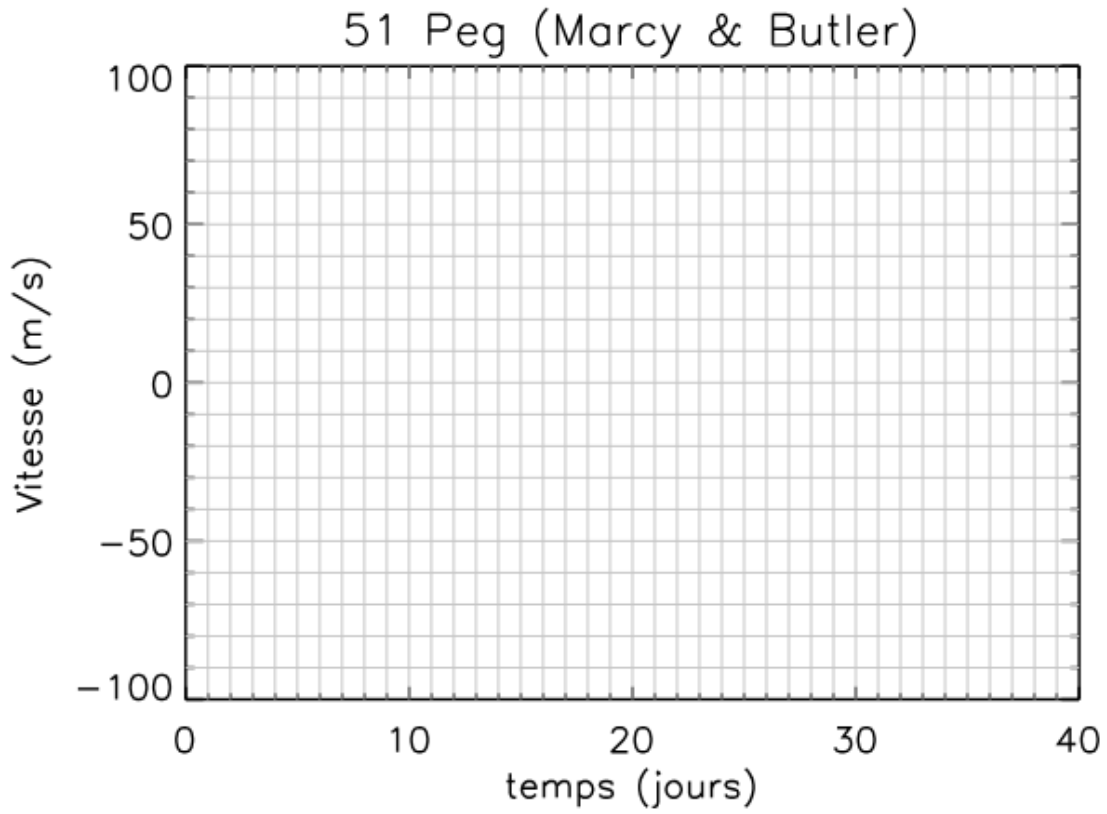

Figure 3: Radial velocity variations of 51 Peg as a function of time.

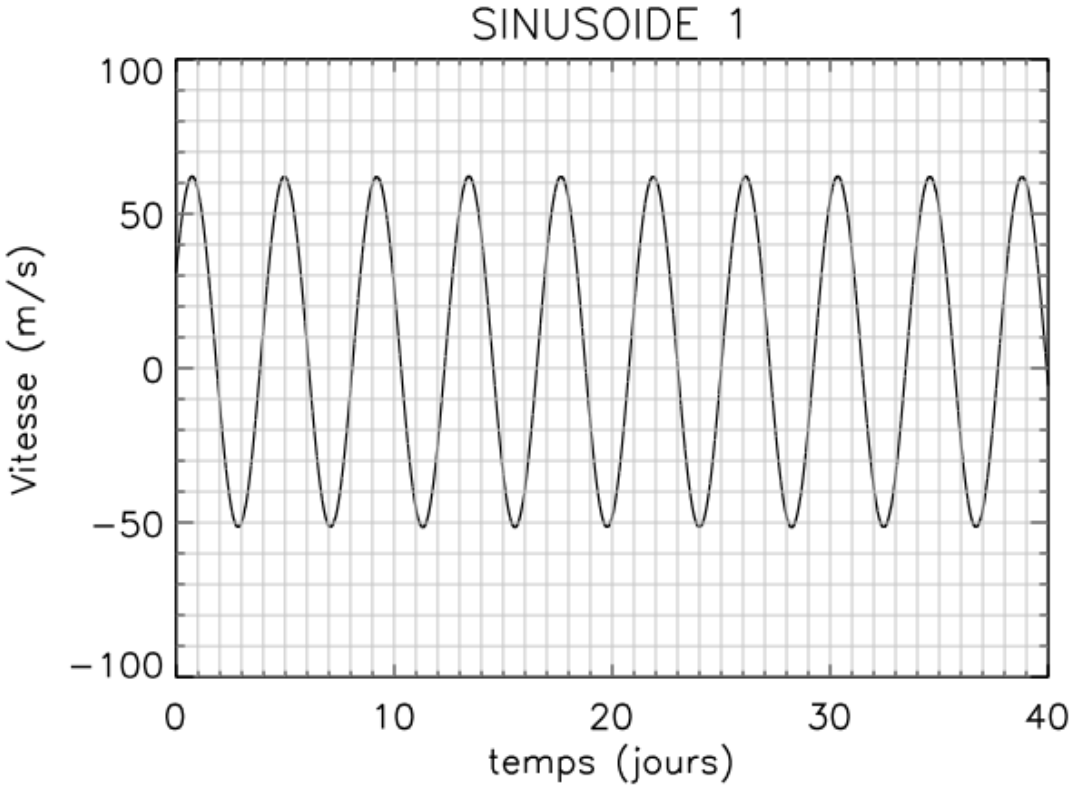

Figure 4: Reference sinusoid #1

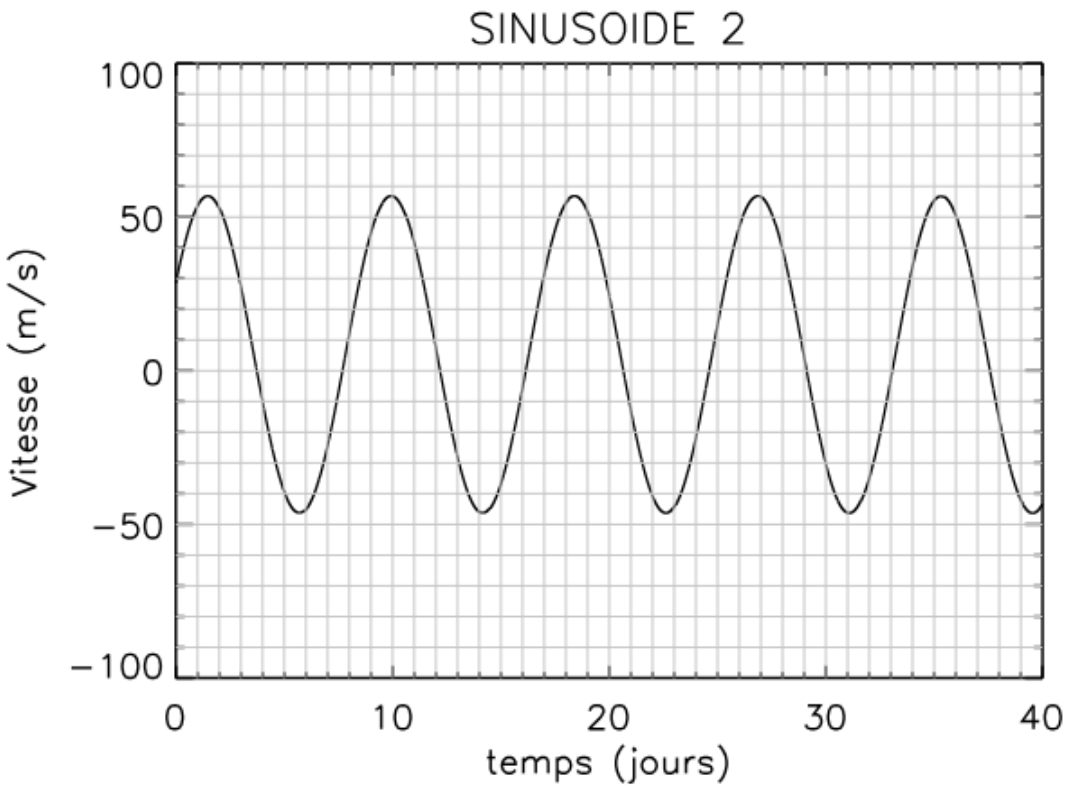

Figure 5: Reference sinusoid #2

3 The exoplanet's parameters

We will now estimate the mass of the planet and its distance from the star. We will consider the system formed by the star and its planet as isolated.

Kepler's laws.

Kepler's three laws describe the motion of planets around their star. They establish the relationship between the orbital period and the star-planet distance. These three laws are valid in the heliocentric reference frame assumed to be Galilean.

Reminders:

- Galilean reference frame: A Galilean reference frame is one in which an isolated object (on which no force is exerted or on which the resultant of forces is zero) is either stationary or in uniform rectilinear translation motion relative to this reference frame. This means that the principle of inertia, set out in Newton's First Law, applies.

- Principle of inertia: In a Galilean frame of reference, the velocity vector of the centre of inertia of a system is constant if and only if the sum of the forces acting on the system is zero.

Kepler's first law: law of orbits

All the planets describe an elliptical motion with the Sun at one of the foci (F in Figure 6).

Reminders about ellipses (Figure 7):

- AC=2a: major axis
- BD: minor axis

- O: centre
- F and F': foci
- a: length of the semi-major axis
- b: length of the semi-minor axis
- c: distance from the centre to the focus
- e: eccentricity = c/a

For an ellipse, the eccentricity is between 0 and 1. See Figure 8 for some examples of ellipses with different eccentricities.

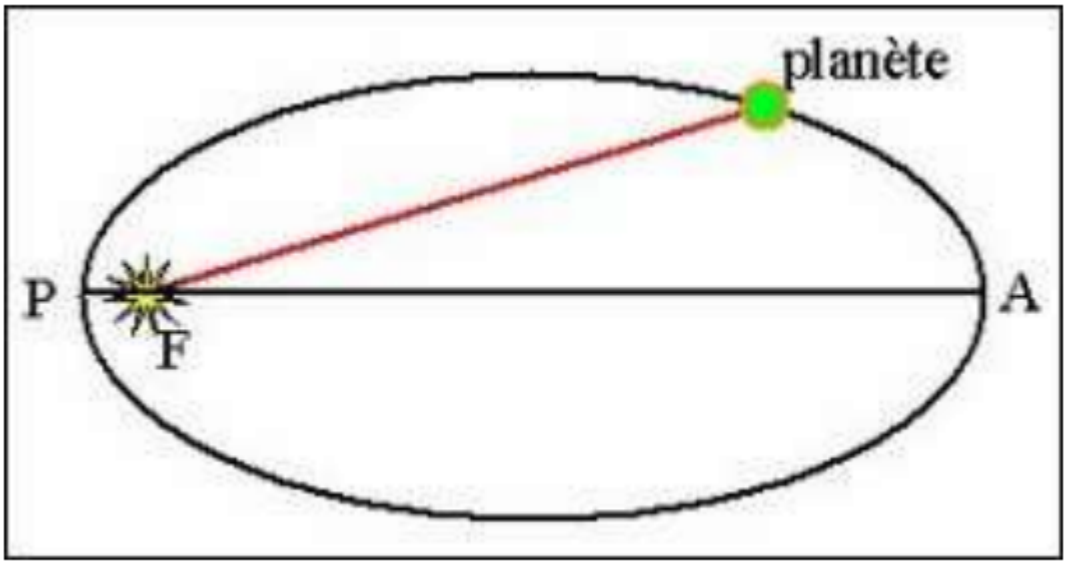

Figure 6: The Sun is located at focus F. P, the perihelion, is the closest point to the Sun. A, the aphelion, is the point furthest from the Sun. The time taken to cover the ellipse once is called the period.

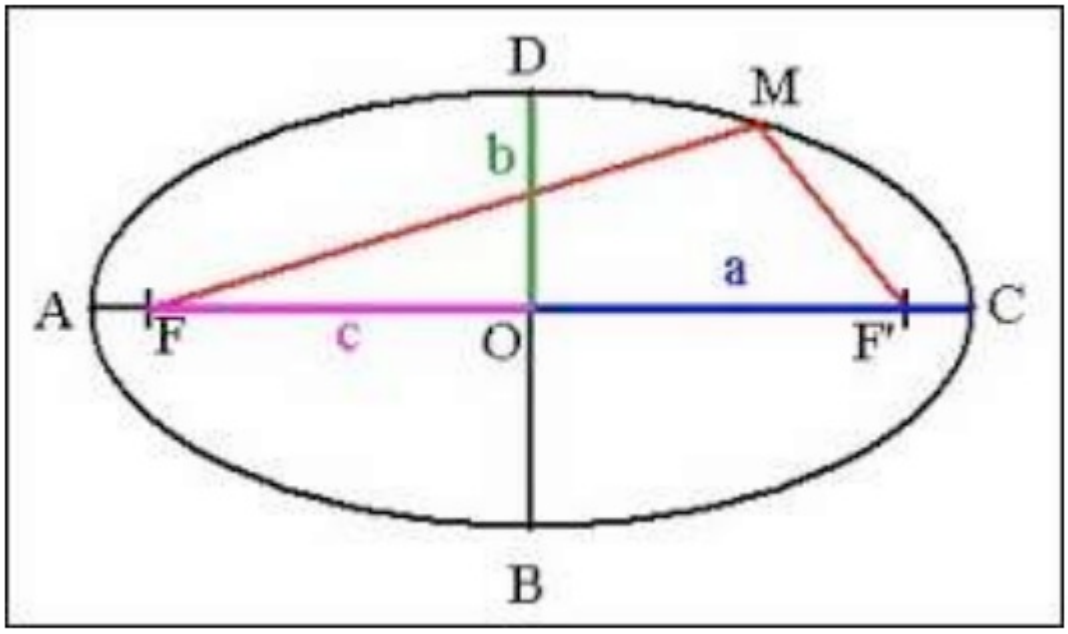

Figure 7: The ellipse is the set of points M in the plane for which the sum of the distances to two fixed points in the same plane (F and F') is a given constant: MF+MF'=2a. The two fixed points F and F' are called foci. Note: a circle can be considered as an ellipse whose two foci coincide.

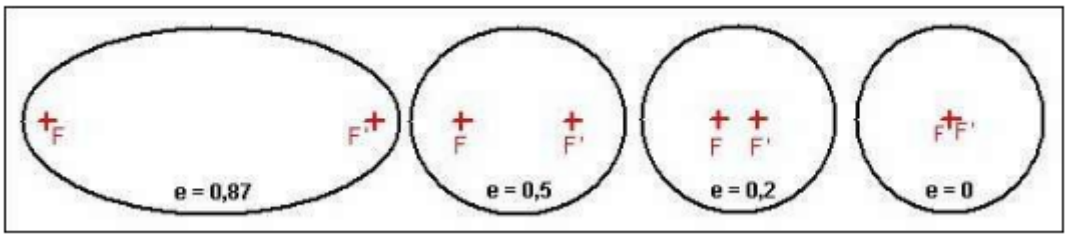

Figure 8: The lower the eccentricity, the closer the ellipse is to the circle.

Kepler's second law: the law of areas
The straight line segment connecting the centres of the star and the planet sweeps over equal areas for equal times (see Figure 9).

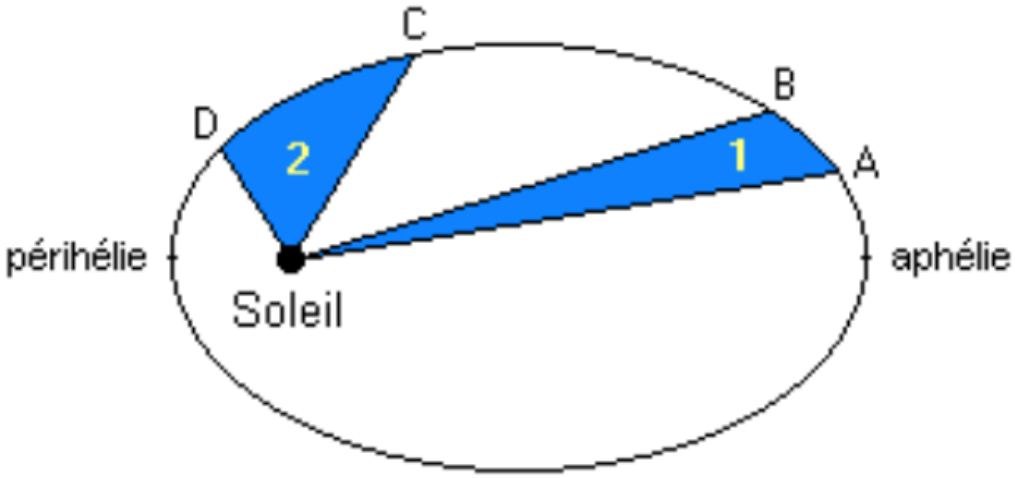

Figure 9: Kepler's second law. Area 1 is equal to area 2 and the arcs AB and CD, of different lengths, are covered in the same time. Consequently, the speed of a planet in its orbit is not constant: the closer the planet is to the Sun, the greater its speed (maximum at perihelion and minimum at aphelion).

Kepler's third law: the law of periods
The ratio between the square of the period of revolution (*T*) of a planet around its star and the cube of the semi-major axis of the elliptical orbit is constant:

$$\frac{T^2}{a^3} = C = \frac{4\pi^2}{\mathcal{G}M_{star}}$$

where *C* is a constant. The value of this constant depends only on the star.

Application of Kepler's third law: star-planet distance
Using Kepler's third law, calculate the distance between the planet and its star, knowing that the mass of the star 51 Peg a is $0.95M_{\odot}$. The result will be given in astronomical units (AU).

Application of Kepler's third law: the exoplanet's mass

We saw in section 1 that the measured radial velocity of the star is of the form:

$$v_{star}(t) = V.\sin\left(2\pi\frac{t}{T} + \varphi\right)$$

The relationship between the measured radial velocity and the velocity of the star in its orbit is:

$$v_{rad}(t) = v_{star}(t)\sin i$$

where *i* is the angle between the normal to the plane of the orbit of the star-planet system and the line of sight (Figure 10). The observer sees the projection of the star's velocity onto the line of sight.

We will now try to define the expression for $K = V \sin i$, which is the amplitude of v_{rad} measured from the curve of radial velocity variations. It is from *K* that we can estimate the mass of the planet.

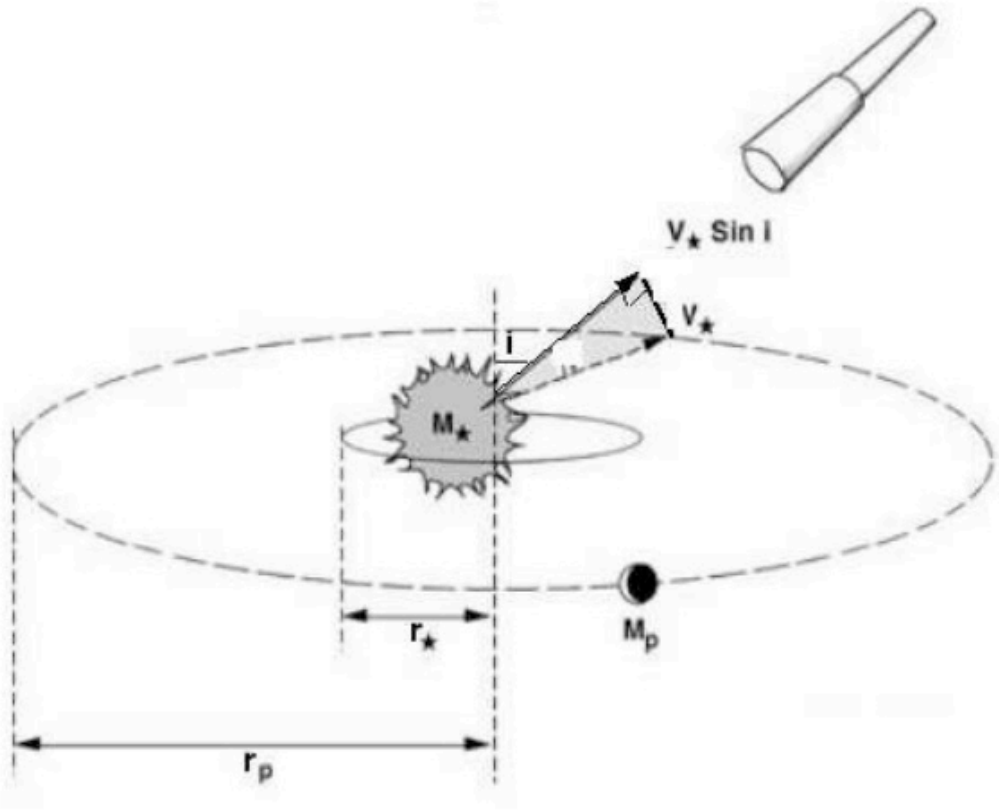

Figure 10: Definition of angle *i*, the angle between the normal to the orbit plane of the star-planet system and the boresight axis

Questions

- Establish the relationship between the masses of the planet and the star (M_{star} and m_{planet}) and their distances from the centre of gravity G (r_{star} and r_{planet}). We will use the definition of the system's centre of gravity.
- Derive the expression for the velocity of the star, V, as a function of m_{planet}, M_{star}, *T* and *a* which is the distance between the star and the planet. It will be assumed that the star is coincident with the centre of gravity so $r_{planet} \sim a$.

- Use Kepler's 3rd law to determine the expression for m_{planet} sin i as a function of K, M M_{star} and T . The mass of the planet will be neglected in front of that of the star.
- Calculate m_{planet} sin i for the planet 51 Peg b. The result will be presented in units of the mass of Jupiter.

Using this method, which is based on variations in the radial velocity of the star, we can deduce not the actual mass of the planet m_{planet} but m_{planet} sin i, where i is the angle through which we see the orbital plane. The actual mass of the planet is therefore unknown, and depends on the angle at which the planetary system is observed, an angle that is not always known (Figure 10).

We therefore obtain a lower value for the mass of the planet, i.e. the mass of the planet cannot be smaller than the value found.

4 Limitations of the method of detecting exoplanets by spectroscopy

Questions:

- Calculate the velocity, V, of our Sun perturbed by:
  1. Jupiter
  2. the Earth
- In 1995, the precision of the instruments was of the order of 15 m/s. Could we detect telluric planets at the same distance from the star as the telluric planets in the Solar System? What about giant planets?
- The current precision of instruments is of the order of 1 m/s (the HARPS spectrograph installed on an ESO 3m60 telescope at La Silla in Chile). Can we now detect telluric planets like those in the Solar System?
- Why won't a planet like Saturn in our solar system be detected before 2025?
- This detection method has a bias. It favours the detection of planets with particular properties. What properties?

References:

- Mayor M., & Queloz D., 1995, Nature, Vol. 378, p. 355.
- “Lumières sur l’Univers https://ufe.obspm.fr/formations/diplomes-d-universite/du-lu-lumieres-sur-l-univers/article/presentation-generale